Fermi Surface Renormalization in Two Spatial Dimensions

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Abstract

We discuss the renormalization induced by interactions of a two-dimensional truncated Fermi surface (FS) model.Using a field theoretical renormalization group method we calculate the critical renormalized physical chemical potential μ_R. We show that it either vanishes or approaches a non-zero value. We argue that the vanishing of μ_R is indicative of a further truncation of the FS we started with and might well represent an insulating spin liquid phase.

Strongly interacting fermion systems continue to attract great interest. This was mainly motivated by the appearance of the high temperature superconductors and by the realization that these materials have, at low and optimal dopings, many anomalous properties. Besides, at zero doping they are antiferromagnetic Mott insulators[1] and it is therefore clear that strong correlation effects play a decisive role in this phenomenon.

Metallic systems are readily characterized by their Fermi Surface(FS) and by their low-lying fermionic excitations. However in one spatial dimension($d = 1$), in the framework of a Tomonaga-Luttinger liquid, the FS is well defined but the low- lying excited states are bosonic modes[2],[3]. The quasiparticles are therefore destroyed by the interactions in $d = 1$. Nevertheless the non-interacting Fermi momentum k_F continues to be a relevant physical parameter and it is non-renormalizable not to violate Luttinger´s theorem [4] [5]. Thus in the Tomonaga-Luttinger liquid particle-hole excitations continue to exist at $2k_F$ and the interacting single-particle Green´s function is such that $\operatorname{Im} G$ shows a change of sign at the unrenormalized k_F.

We want to consider in this work what should happen to the FS of strongly interacting fermions in $d = 2$. The importance of performing a proper FS renormalization was also emphasized recently by Kopietz and Busche[6] who treat the Fermi surface as a renormalization group(RG) fixed point manifold. Earlier on the the renormalization of the Fermi surface for $2d$ electrons near a van Hove singularity was discussed by Gonzalez, Guinea and Vozmediano[7] Here we approach this problem using a field theoretical RG method which allows us to deal with these questions in a direct way. In higher dimensions the renormalization of FS does not necessarily imply the violation of Luttinger´s theorem[8].

To initiate our discussion we consider one-particle fermionic states described by the renormalized lagrangian density

$$L = Z\left(x\right)\psi_\sigma^\dagger\left(x\right)\left[i\partial_t + \frac{\nabla^2}{2} + \mu_0\left(x\right)\right]\psi_\sigma\left(x\right) +$$

$$- \int_{\overline{x}} U_0\left(x - \overline{x}\right) Z\left(x\right) Z\left(\overline{x}\right) \psi_\uparrow^\dagger\left(x\right) \psi_\downarrow^\dagger\left(\overline{x}\right) \psi_\downarrow\left(\overline{x}\right) \psi_\uparrow\left(x\right) \qquad (1)$$

where μ_0 is the bare chemical potential, U_0 is the bare coupling function and $Z\left(x\right)$ is a local charge renormalization parameter. Here the word bare is used in a field theory context and does not mean non-interacting. Since L is exact the Green´s function which result from it are already renormalized. The spatial dependence of both U_0 and Z in our model reflect the fact that we use a truncated $2d$ Fermi surface which consists of four symmetrical patches centred around $(\pm k_F, 0)$ and $(0, \pm k_F)$ in **k**-space as our unrenormalized FS. As we show in Figure 1 inside each patch in our model there are flat sectors in the border regions which are continuously connected to a curved arc around its center[9]. This model was used before to describe two-dimensional quasiparticles states sandwiched by one-particle states with one-dimensional energy dispersion

law to simulate the "cold" and "hot" spots[10][11] respectively. As a result of the momentum space anisotropy the bare parameters become local functions of momenta and the regularization is no longer isotropic and simple.

Following renormalization theory[12] let us rewrite our lagrangian density in a more convenient form:

$$L = \psi_\sigma^\dagger \left(i\partial_t + \frac{\nabla^2}{2} + \mu\left(\omega\right) \right) \psi_\sigma - \int_{\overline{x}} U\left(x - \overline{x}\right) \psi_\uparrow^\dagger\left(x\right) \psi_\downarrow^\dagger\left(\overline{x}\right) \psi_\downarrow\left(\overline{x}\right) \psi_\uparrow\left(x\right)$$

$$+\psi_\sigma^\dagger \left[\Delta Z\left(x\right) \left(i\partial_t + \frac{\nabla^2}{2} \right) + \Delta\mu_0\left(x\right) \right] \psi_\sigma +$$

$$- \int_{\overline{x}} \Delta U_0\left(x - \overline{x}\right) \psi_\uparrow^\dagger\left(x\right) \psi_\downarrow^\dagger\left(\overline{x}\right) \psi_\downarrow\left(\overline{x}\right) \psi_\uparrow\left(x\right) \tag{2}$$

where $\mu\left(\omega\right) = \frac{1}{2} k_F^2\left(\omega\right)$ is the renormalized chemical potential parameter and ω is an energy scale parameter. Besides, we have that

$$\Delta\mu_0\left(x\right) = Z\left(x\right) \mu_0\left(x\right) - \frac{1}{2} k_F^2\left(\omega\right), \tag{3}$$

$$\Delta Z\left(x\right) = Z\left(x\right) - 1 \tag{4}$$

and finally

$$\Delta U_0\left(x - \overline{x}\right) = U_0\left(x - \overline{x}\right) Z\left(x\right) Z\left(\overline{x}\right) - U\left(x - \overline{x}\right) \tag{5}$$

Here $\mu\left(\omega\right)$ is not directly associated with the physical Fermi momentum which is associated with the number density. In fact as we will see later $\mu\left(\omega\right) \to 0$ as we let $\omega \to 0$. Let us now use L to calculate perturbatively the renormalized one-particle irreducible function $\Gamma_R^{(2)}\left(\mathbf{p}, p_0; \omega\right)$ for $\mathbf{p}$ in the vicinity of the FS point $\left(\Delta\left(\omega\right), -k_F\left(\omega\right) + \frac{\Delta^2\left(\omega\right)}{2k_F\left(\omega\right)} \right)$. Up to order $O\left(U^2\right)$ we find

$$\Gamma_{R\uparrow}^{(2)}\left(\mathbf{p}, p_0; \omega\right) = p_0 + k_F\left(\omega\right) \left(p_y + k_F\left(\omega\right) + \frac{\Delta^2\left(\omega\right)}{2k_F\left(\omega\right)} \right) - \Sigma_{R\uparrow}\left(\mathbf{p}, p_0; \omega\right), \tag{6}$$

where the renormalized self-energy $\Sigma_{R\uparrow}$ is determined by the Feynman diagrams shown in Figure 2.

The last two diagrams of $-i\Sigma_{R\uparrow}$ are originated by the counterterms of the renormalized lagrangian and will be defined in detail later on. Our convention here is to associate the crossed dot with the bare coupling function $\Delta U_0\left(\mathbf{p}_1, \mathbf{p}_4\right)$ given by

$$\Delta U_0\left(\mathbf{p}_1,\mathbf{p}_4\right)=Z\left(\mathbf{p}_1\right)Z\left(\mathbf{p}_4\right)U_0\left(\mathbf{p}_1,\mathbf{p}_4\right)-U$$

$$\cong\frac{4\lambda\left(\omega\right)}{2\pi^2 k_F\left(\omega\right)}U^2\ln\left(\frac{\Omega}{\omega}\right)\delta_{\mathbf{p}_1+\mathbf{p}_2=\mathbf{0}}+$$

$$-\frac{\lambda\left(\omega\right)-\Delta\left(\omega\right)}{2\pi^2 k_F\left(\omega\right)}U^2\ln\left(\frac{\Omega}{\omega}\right)\delta_{\mathbf{p}_1=\mathbf{p}_3}\delta_{\mathbf{p}_1=\mathbf{p}_4=\left(0,-2k_F\left(\omega\right)+\frac{\Delta^2\left(\omega\right)}{k_F\left(\omega\right)}\right)}+\dots \qquad (7)$$

As a result of this it turns out that diagram (d) reduces to

$$\Sigma_{R\uparrow}^{(d)}\left(\mathbf{p},p_0;\omega\right)=\frac{U^2\left(\omega\right)}{2\pi^4}\lambda\left(\omega\right)\left(\lambda\left(\omega\right)-\Delta\left(\omega\right)\right)\left(\frac{3\lambda\left(\omega\right)+\Delta\left(\omega\right)}{k_F\left(\omega\right)}\right)\ln\left(\frac{\Omega}{\omega}\right)+\dots \qquad (8)$$

Since diagrams (a) and (b) produce[9]

$$\Sigma_{R\uparrow}^{(a+b)}\left(\mathbf{p},p_0;\omega\right)=\frac{2\lambda^2\left(\omega\right)}{\pi^2}U-\frac{3U^2}{64\pi^4}\left(\frac{\lambda\left(\omega\right)-\Delta\left(\omega\right)}{k_F\left(\omega\right)}\right)^2\left(p_0+k_F\left(\omega\right)\left(p_y+\right.\right.$$

$$\left.\left.+k_F\left(\omega\right)+\frac{\Delta^2\left(\omega\right)}{2k_F\left(\omega\right)}\right)\right)\cdot$$

$$\cdot\left[\ln\left(\frac{\Omega+p_0-i\delta}{-k_F\left(\omega\right)\left(p_y+k_F\left(\omega\right)+\frac{\Delta^2\left(\omega\right)}{2k_F\left(\omega\right)}\right)+p_0-i\delta}\right)\right.$$

$$\left.+\ln\left(\frac{\Omega-p_0-i\delta}{k_F\left(\omega\right)\left(p_y+k_F\left(\omega\right)+\frac{\Delta^2\left(\omega\right)}{2k_F\left(\omega\right)}\right)-p_0-i\delta}\right)\right]+\dots \qquad (9)$$

and since diagram (c) gives simply

$$\Sigma_{R\uparrow}^{(c)}\left(\mathbf{p},p_0;\omega\right)=-\Delta Z\left(\frac{\overline{p}}{\omega};\omega\right)\left(p_0-\frac{p^2}{2}\right)-\Delta\mu_0\left(\frac{\overline{p}}{\omega};\omega\right), \qquad (10)$$

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with $\overline{p}\left(\omega\right) = k_F\left(\omega\right)\left(p_y + k_F\left(\omega\right) + \frac{\Delta^2(\omega)}{2k_F(\omega)}\right)$ the renormalized $\Gamma^{(2)}_{R\uparrow}\left(\mathbf{p}, p_0; \omega\right)$ in the vicinity of $\mathbf{p}^* = \left(\Delta\left(\omega\right), -k_F\left(\omega\right) + \frac{\Delta^2(\omega)}{2k_F(\omega)}\right)$ becomes

$$\Gamma^{(2)}_{R\uparrow}\left(\mathbf{p}, p_0; \omega\right) = \left(p_0 + k_F\left(\omega\right)\left(p_y + k_F\left(\omega\right) + \frac{\Delta^2\left(\omega\right)}{2k_F\left(\omega\right)}\right)\right)\left[1 + \right.$$

$$\frac{3U^2}{64\pi^4}\left(\frac{\lambda\left(\omega\right) - \Delta\left(\omega\right)}{k_F\left(\omega\right)}\right)^2\left(\ln\left(\frac{\Omega - p_0 - i\delta}{k_F\left(\omega\right)\left(p_y + k_F\left(\omega\right) + \frac{\Delta^2(\omega)}{2k_F(\omega)}\right) - p_0 - i\delta}\right)\right.$$

$$\left.\left. + \ln\left(\frac{\Omega + p_0 - i\delta}{-k_F\left(\omega\right)\left(p_y + k_F\left(\omega\right) + \frac{\Delta^2(\omega)}{2k_F(\omega)}\right) + p_0 - i\delta}\right)\right)\right] - \frac{2\lambda^2\left(\omega\right)}{\pi^2}U$$

$$+ \Delta Z\left(\frac{\overline{p}}{\omega}; \omega\right)\left(p_0 - \frac{p^2}{2}\right) + \Delta\mu_0\left(\frac{\overline{p}}{\omega}; \omega\right)$$

$$- \frac{U^2\left(\omega\right)}{2\pi^4}\lambda\left(\omega\right)\left(\lambda\left(\omega\right) - \Delta\left(\omega\right)\right)\left(\frac{3\lambda\left(\omega\right) + \Delta\left(\omega\right)}{k_F\left(\omega\right)}\right)\ln\left(\frac{\Omega}{\omega}\right) + ... \qquad (11)$$

To determine ΔZ and $\Delta\mu_0$ we now define $\Gamma^{(2)}_R$ such that $\Gamma^{(2)}_{R\uparrow}\left(\mathbf{p}*, \omega; \omega\right) = \omega$, with $\omega \approx 0$. From this prescription we find

$$\Delta Z\left(\frac{\overline{p}}{\omega} = 0; \omega\right) = -\frac{3U^2}{32\pi^4}\left(\frac{\lambda\left(\omega\right) - \Delta\left(\omega\right)}{k_F\left(\omega\right)}\right)^2\ln\left(\frac{\Omega}{\omega}\right) + ... \qquad (12)$$

and, similarly,

$$\mu_0 = \frac{1}{2}k_F^2\left(\omega\right) + \frac{2\lambda^2\left(\omega\right)}{\pi^2}U\left(\omega\right)\left[1 + \right.$$

$$\left. + \frac{U\left(\omega\right)}{2\pi^2}\left(\frac{\lambda\left(\omega\right) - \Delta\left(\omega\right)}{2\lambda\left(\omega\right)}\right)\left(\frac{3\lambda\left(\omega\right) + \Delta\left(\omega\right)}{k_F\left(\omega\right)}\right)\ln\left(\frac{\Omega}{\omega}\right) + ...\right] \qquad (13)$$

If we now replace these results back into the equation for $\Gamma^{(2)}_{R\uparrow}$, we find for $\mathbf{p} \approx \mathbf{p}^*$ and $p_0 \approx \omega$

$$\Gamma_{R\uparrow}^{(2)}\left(\mathbf{p}, p_0; \omega\right) = \left(p_0 + k_F\left(\omega\right)\left(p_y + k_F\left(\omega\right) + \frac{\Delta^2\left(\omega\right)}{2k_F\left(\omega\right)}\right)\right)[1+$$

$$\frac{3U^2}{64\pi^4}\left(\frac{\lambda\left(\omega\right) - \Delta\left(\omega\right)}{k_F\left(\omega\right)}\right)^2 \cdot$$

$$\cdot\left(\ln\left(\frac{\omega}{k_F\left(\omega\right)\left(p_y + k_F\left(\omega\right) + \frac{\Delta^2\left(\omega\right)}{2k_F\left(\omega\right)}\right) - p_0 - i\delta}\right)\right.$$

$$\left.+\ln\left(\frac{\omega}{-k_F\left(\omega\right)\left(p_y + k_F\left(\omega\right) + \frac{\Delta^2\left(\omega\right)}{2k_F\left(\omega\right)}\right) + p_0 - i\delta}\right)\right) + ...] \tag{14}$$

Consequently at $p_0 = 0$ and $\mathbf{p} \approx \mathbf{p}^*$ the renormalized self-energy is such that

$$\operatorname{Im}\Sigma_{R\uparrow}\left(\mathbf{p}, p_0 = 0; \omega\right) = -\frac{3U^2}{64\pi^4}\left(\frac{\lambda\left(\omega\right) - \Delta\left(\omega\right)}{k_F\left(\omega\right)}\right)^2 k_F\left(\omega\right)\left(p_y +\right.$$

$$\left.+k_F\left(\omega\right) + \frac{\Delta^2\left(\omega\right)}{2k_F\left(\omega\right)}\right)\left[\frac{\pi}{2}\theta\left(\overline{p}\right) + \theta\left(-\overline{p}\right)\right] + ... \tag{15}$$

Thus the imaginary part of the renormalized Green´s function $\operatorname{Im} G$ changes sign at $\overline{p}\left(\omega\right) = 0$. Since from our previous result we also have that

$$\operatorname{Re}\Sigma_{R\uparrow}\left(\overline{p} = 0, p_0 = 0; U\left(\omega\right); \omega\right) = 0 \tag{16}$$

we can define a dimensionless renormalized chemical potential $\overline{\mu}\left(\omega\right)$ through

$$\mu_0 = \omega Z_\mu\left(\omega\right)\overline{\mu}\left(\omega\right) = Z_\mu\left(\omega\right)\frac{1}{2}k_F^2\left(\omega\right), \tag{17}$$

where, using our perturbative results

$$Z_\mu\left(\omega\right) = 1 + \frac{U\left(\omega\right)}{\pi^2}\left(\frac{2\lambda\left(\omega\right)}{k_F\left(\omega\right)}\right)^2[1+$$

$$\frac{U\left(\omega\right)}{2\pi^2}\left(\frac{\lambda\left(\omega\right) - \Delta\left(\omega\right)}{2\lambda\left(\omega\right)}\right)\left(\frac{3\lambda\left(\omega\right) + \Delta\left(\omega\right)}{k_F\left(\omega\right)}\right)\ln\left(\frac{\Omega}{\omega}\right) + ...] \tag{18}$$

To go beyond this two-loop result we can invoke the renormalization group *(RG)*. In this way, since $\omega d\mu_0/d\omega = 0$ it follows immediately that

$$\omega \frac{d\overline{\mu}(\omega)}{d\omega} = -\left(1+\gamma_\mu\right)\overline{\mu}(\omega), \tag{19}$$

where $\gamma_\mu = d\ln Z_\mu(\omega)/d\ln\omega$.

In general we have that $Z_\mu = Z_\mu\left(\omega; \frac{\lambda(\omega)}{k_F(\omega)}; \frac{\Delta(\omega)}{k_F(\omega)}; U(\omega)\right)$. To perform a full calculation of γ_μ is a difficult problem. However if we assume that the renormalized coupling and the ratios of the Fermi surface parameters are fixed point values we can use our perturbative results to find

$$\gamma_\mu = -\frac{U^{*2}}{\pi^4}\left(\frac{\lambda-\Delta}{k_F}\right)^*\left(\frac{\lambda}{k_F}\right)^*\left(\frac{3\lambda+\Delta}{k_F}\right)^* = -\gamma_\mu^* \tag{20}$$

If we then replace this into the *RG* equation for $\overline{\mu}(\omega)$ we find

$$\overline{\mu}(\omega) = \overline{\mu}(\Omega)\left(\frac{\omega}{\Omega}\right)^{-\left(1-\gamma_\mu^*\right)} \tag{21}$$

Hence the renormalized chemical potential parameter $\mu(\omega) = \frac{1}{2}k_F^2(\omega) \sim \Omega\left(\frac{\omega}{\Omega}\right)^{\gamma_\mu^*}$, for a non-trivial fixed point U^*, is nullified by the anomalous dimension γ_μ^* when $\omega \to 0$. Note that even if $\mu(\omega) \to 0$ the physical potential which we define more generally as $\mu_R\left(\frac{\mu}{\omega}; U(\omega)\right)$ might not be. To test this assumption we use the ansatz[13]

$$\mu_R\left(\frac{\mu}{\omega}; U(\omega)\right) = \mu f\left(\exp t; U(\omega)\right) \tag{22}$$

with $t = \ln\left(\frac{\mu}{\omega}\right)$. Since $\mu_R\left(\frac{\mu}{\omega}; U(\omega)\right)$ satisfies the *RG* equation

$$\left[\omega\frac{\partial}{\partial\omega} + \beta(U)\frac{\partial}{\partial U} + \omega\frac{\partial\ln\mu(\omega)}{\partial\omega}\mu\frac{\partial}{\partial\mu}\right]\mu_R\left(\frac{\mu}{\omega}; U(\omega)\right) = 0 \tag{23}$$

it then follows that the function f is determined by

$$\left[-\frac{\partial}{\partial t} + \widetilde{\beta}(U)\frac{\partial}{\partial U} + \widetilde{\gamma}_\mu(U)\right]f\left(\exp t; U\right) = 0 \tag{24}$$

where

$$\widetilde{\beta}(U) = \frac{\beta(U)}{1 - \omega\frac{\partial\ln\mu(\omega)}{\partial\omega}} \tag{25}$$

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and

$$\widetilde{\gamma}_\mu\left(U\right) = \frac{\omega \frac{\partial \ln \mu(\omega)}{\partial \omega}}{1 - \omega \frac{\partial \ln \mu(\omega)}{\partial \omega}} \tag{26}$$

The general solution for f is of the form

$$f\left(\exp t; U\right) = f\left(1; U\left(t; U\right)\right) \exp \int_0^t d\bar{t} \widetilde{\gamma}_\mu \left(U\left(\bar{t}; U\right)\right) \tag{27}$$

where

$$\frac{\partial U\left(t; U\right)}{\partial t} = \widetilde{\beta}\left(U\left(t; U\right)\right), \tag{28}$$

with $U\left(t = 0; U\right) = 0$. If we consider the limit $\mu \to 0$ or $t \to -\infty$, assuming again that the physical system is brought to a critical condition with $U \to U^*$, the physical chemical potential reduces to

$$\lim_{\mu \to 0} \mu_R \left(\frac{\mu}{\omega}; U\left(\omega\right)\right) = \mu f\left(1; U^*\right) \left(\frac{\mu}{\omega}\right)^{\frac{\gamma_\mu^*}{1-\gamma_\mu^*}} \sim \mu^{\frac{1}{1-\gamma_\mu^*}} \tag{29}$$

Therefore only if $1 > \gamma_\mu^*$, $\mu_R \left(\frac{\mu}{\omega}; U^*\right) \to 0$ when $\mu \to 0$. Note that the existence of a non-trivial infrared(IR) stable U^* is a necessary condition for a non-zero μ_R. Since we already proved the existence of such fixed points elsewhere[9] a non-zero μ_R is clearly attainable for appropriate renormalized FS parameters. Moreover if we use our earlier estimate $U^* = \frac{8\pi^2}{3} \left(\frac{k_F}{\lambda - \Delta}\right)^*$, the latter inequality becomes $\left(\frac{k_F}{3\lambda + \Delta}\right)^* > \frac{64}{9} \left(\frac{\lambda}{\lambda - \Delta}\right)^*$. Thus the FS will suffer a further truncation in $\mathbf{k}$-space wherever this inequality is satisfied. This result is very suggestive in view of the observation that in the pseudogap phase of the underdoped high temperature superconductors the FS is indeed truncated and there should be an insulating phase associated with the momentum space region where the charge gap acquires a non-zero value. That insulating state might well be the spin liquid proposed by Furukawa, Rice and Salmhofer earlier on[14].

In conclusion we showed that in two spatial dimensions the Fermi surface is renormalized by interactions. If the physical system approaches criticality with non-trivial IR coupling fixed points the renormalized physical potential μ_R can either vanish or acquire a non-zero value. The vanishing of μ_R is indicative that the Fermi surface can suffer a further truncation and the resulting insulating state might well be an insulating spin liquid phase.

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Figure 1: Truncated $2d$ Fermi Surface.

Figure 2: Self-energy $\Sigma_{R\uparrow}$ diagrams up to two-loops.

Figure 1:

Figure 2:

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